Understanding Reasoning in Elementary School Math
Selected Quotes

"Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true. Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion." (p. 6)

Alberta Education. (2007/2014). *Mathematics, kindergarten to grade 9 program of studies*. Edmonton, AB: Author.

"Being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and—with different expectations of sophistication—at all grade levels, students should see and expect that mathematics makes sense. … Reasoning and proof cannot simply be taught in a single unit on logic … [and] should be a consistent part of students' mathematical experience in prekindergarten through grade 12." (p. 56)

"Young children will express their conjectures and describe their thinking in their own words and often explore them using concrete materials and examples. Students at all grade levels should learn to investigate their conjectures using concrete materials, calculators and other tools, and increasingly through the grades, mathematical representations and symbols. They also need to learn to work with other students to formulate and explore their conjectures and to listen to and understand conjectures and explanations offered by classmates." (p. 57)

"Along with making and investigating conjectures, students should learn to answer the question, Why does this work? Children in the lower grades will tend to justify general claims using specific case. … By the upper elementary grades, justifications should be more general and can draw on other mathematical results. In high school, students should be expected to construct relatively complex chains of reasoning and provide mathematical reasons. To help students develop and justify more-general conjectures and also to refute conjectures, teachers can ask, 'Does this always work? Sometimes? Never? Why?' This extension to general cases draws on more-sophisticated mathematical knowledge that should build up over the grades." (p. 58)

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

"Practices related to reasoning include expectations that students will (*a*) reason abstractly and quantitatively, (*b*) construct viable arguments and critique the reasoning of others, (*c*) look for and make use of structure, and (*d*) look for and express regularity in repeated reasoning." (p. 8)

"**Big Idea**. Mathematical reasoning is an evolving process of conjecturing, generalizing, investigating *why*, and developing and evaluating arguments.

Conjecturing and Generalizing

Essential Understanding 1. Conjecturing involves reasoning about mathematical relationships to develop statements that are tentatively thought to be true but are not know to be true. These statements are called *conjectures*.

Essential Understanding 2. Generalizing involves identifying commonalities across cases or extending the reasoning beyond the range in which it originated.

Essential Understanding 3. Generalizing involves identifying the application of the generalization by recognizing the relevant domain.

Essential Understanding 4. Conjecturing and generalizing involve using and clarifying the meaning of terms, symbols, and representations.

Investigating *Why*

Essential Understanding 5. Mathematical reasoning involves investigating various potential factors that may explain *why* a generalization is true or false.

Justifying and Refuting

Essential Understanding 6. A mathematical justification is a logical argument based on already-understood ideas.

Essential Understanding 7. A mathematical refutation involves demonstrating that a particular statement is false.

Essential Understanding 8. Justifying and refuting involve evaluating the validity of arguments.

Essential Understanding 9. A valid mathematical justification for a general statement is *not* an argument based on authority, perception, popular consensus, or examples." (p. 12)

Lannin, J., Ellis, A. B., & Elliott, R. (2011). *Developing essential understanding of mathematical reasoning for teaching mathematics in prekindergarten-grade 8*. Reston, VA: NCTM.

"Three characteristics … central [to learning to prove]: conjecturing, leaving the criteria for a correct solution up to the students, and expectations for communication. Because the students were making conjectures, they had to have a certain level of understanding of the problem, and they had a personal investment in determining whether or not their conjectures were correct. Because they had to develop their own methods and criteria for verifying their conjectures, they explored ways of reasoning and arguing. Because they had to communicate their arguments to their peers and according to social expectations, they had to be careful in how they formulated their reasoning, they had to be able to explain it clearly, and they had to be willing to reformulate their reasoning if their expression was not satisfactory. These three characteristics put an emphasis on reasoning and on arguing." (p. 146)

Reid, D. A., & Zack, V. (2009). Aspects of teaching proving in upper elementary school. In D. A. Stylianou, M. L. Blanton & E. J. Knuth (Eds.), *Teaching and learning proof across the grades: A k-16 perspective* (pp. 133-146). New York: Routledge.